Online Technical Appendix (not for publication)

Section 1. Individual's Maximization Problem in the Absence of Cooperation

$$\max_{N_i^j, N_i^k, z_i} U_i(x_i, z_i, z_j, z_k) \text{ subject to } x_i = Y_i\left(1 - N_i^j - N_i^k, N_j^i, N_k^i; A_i\right) - p_i z_i$$

$$\text{where } N_s^i = N_s^{i*}(N_i^s, N_{-s}^s; \mu_s, p_s, A_s) \text{ and } z_s = z_s^*\left(N_i^s, N_j^s, N_k^s; \mu_s, p_s, A_s\right) \text{ for } s = j, k$$

First order conditions:

$$\frac{\partial U_i}{\partial z_s}\frac{dz_s^*}{dN_i^s} - \frac{\partial U_i}{\partial x_i}\left(\frac{\partial Y_i}{\partial N_i^i} - \frac{\partial Y_i}{\partial N_s^i}\frac{dN_s^{i*}}{dN_i^s}\right) = 0 \text{ for } s = j, k \text{ and } \frac{\partial U_i}{\partial z_i} - p_i\frac{\partial U_i}{\partial x_i} = 0$$

Let D denote the determinant of the Hessian, and define its elements as

$$s_{11} = \frac{\partial U_i}{\partial x_i}\left[\frac{\partial^2 Y_i}{\partial {N_i^i}^2} + \frac{\partial^2 Y_i}{\partial {N_j^i}^2}\left(\frac{dN_j^{i*}}{dN_i^j}\right)^2\right]$$

$$s_{12} = \frac{\partial^2 U_i}{\partial z_j \partial z_k}\frac{dz_j^*}{dN_i^j}\frac{dz_j^*}{dN_i^k} + \frac{\partial U_i}{\partial x_i}\left[\frac{\partial^2 Y_i}{\partial {N_i^i}^2} + \frac{\partial^2 Y_i}{\partial N_i^j \partial N_i^k}\frac{dN_j^{i*}}{dN_i^j}\frac{dN_k^{i*}}{dN_i^k}\right]$$

$$s_{13} = \frac{\partial^2 U_i}{\partial z_j \partial z_i}\frac{dz_j^*}{dN_i^j} - \frac{\partial^2 U_i}{\partial x_i \partial z_j}\left(\frac{\partial Y_i}{\partial N_i^i} - \frac{\partial Y_i}{\partial N_j^i}\frac{dN_j^{i*}}{dN_i^j}\right)$$

$$s_{22} = \frac{\partial U_i}{\partial x_i}\left[\frac{\partial^2 Y_i}{\partial {N_i^i}^2} + \frac{\partial^2 Y_i}{\partial {N_k^i}^2}\left(\frac{dN_k^{i*}}{dN_i^k}\right)^2\right] < 0$$

$$s_{23} = \frac{\partial^2 U_i}{\partial z_k \partial z_i}\frac{dz_k^*}{dN_i^k} - \frac{\partial^2 U_i}{\partial x_i \partial z_k}\left(\frac{\partial Y_i}{\partial N_i^i} - \frac{\partial Y_i}{\partial N_k^i}\frac{dN_k^{i*}}{dN_i^k}\right)$$

$$s_{33} = \frac{\partial^2 U_i}{\partial {z_i}^2} - p_i\frac{\partial^2 U_i}{\partial x_i \partial z_i}$$

Corollary 1. Altruistic players will engage in a greater degree of exchange behavior even when no cooperative agreement is reached, all else equal

Define $\frac{\partial U_i}{\partial z_s} = \theta_s \frac{\partial U_i}{\partial z_i}$. Then for $\theta_s > 0$,

$$\frac{dN_i^j}{d\theta_j} = -\frac{1}{D}\frac{\partial U_i}{\partial z_i}\frac{dz_j^*}{dN_i^j}(s_{22}s_{33} - {s_{23}}^2) > 0 \; and \; \frac{dN_i^k}{d\theta_k} = -\frac{1}{D}\frac{\partial U_i}{\partial z_i}\frac{dz_k^*}{dN_i^k}(s_{11}s_{33} - {s_{13}}^2) > 0$$

If all goods are separable in utility, then

$$\frac{dz_i}{d\theta_j} = -\frac{1}{D}\frac{\partial U_i}{\partial z_i}\frac{dz_j^*}{dN_i^j}(s_{12}s_{23} - s_{13}s_{22}) = 0 \; and \; \frac{dz_i}{d\theta_k} = \frac{1}{D}\frac{\partial U_i}{\partial z_i}\frac{dz_k^*}{dN_i^k}(s_{11}s_{23} - s_{12}s_{13}) = 0$$

In the absence of explicit cooperation, labor allocations to other players are increasing in the degree of altruism.

For $\theta_s = 0$, $\partial U_i / \partial z_s = 0$ and, since player s derives no utility from z_i, he/she has no incentive to reciprocate any labor sharing. Therefore, $dN_s^{i^*} / dN_i^s = 0$. When both of the these conditions hold, first order condition becomes

$$\frac{\partial U_i}{\partial x_i} \frac{\partial Y_i}{\partial N_i^i} = 0$$

Clearly, no interior solution exists, and therefore it must be the case that $N_i^s = 0$ when $\theta_s = 0$. ▪

Corollary 2. Even with labor sharing between players, the allocation of their labor inputs will not be efficient in the absence of explicit cooperation.

To consider the case of altruistic preferences, rewrite the first order condition as

$$\frac{\partial U_i}{\partial z_s} \frac{dz_s^*}{dY_s} \left(\frac{\partial Y_s}{\partial N_i^s} + \frac{\partial Y_s}{\partial N_s^s} \frac{dN_s^s}{dN_i^s} + \frac{\partial Y_s}{\partial N_{-s}^s} \frac{dN_{-s}^s}{dN_i^s} \right) = \frac{\partial U_i}{\partial x_i} \left(\frac{\partial Y_i}{\partial N_i^i} - \frac{\partial Y_i}{\partial N_s^i} \frac{dN_s^{i^*}}{dN_i^s} \right)$$

Rearranging terms yields

$$\frac{\partial Y_i}{\partial N_i^i} = \left(\frac{\partial U_i}{\partial z_s} \frac{dz_s^*}{dY_s} \Big/ \frac{\partial U_i}{\partial x_i} \right) \left(\frac{\partial Y_s}{\partial N_i^s} + \frac{\partial Y_s}{\partial N_s^s} \frac{dN_s^s}{dN_i^s} + \frac{\partial Y_s}{\partial N_{-s}^s} \frac{dN_{-s}^s}{dN_i^s} \right) + \frac{\partial Y_i}{\partial N_s^i} \frac{dN_s^{i^*}}{dN_i^s}.$$

In order for the marginal product of player *i*'s labor to be equalized across plots such that

$$\frac{\partial Y_i}{\partial N_i^i} = \frac{\partial Y_s}{\partial N_i^s},$$

several conditions must hold:

(i) the marginal rate of transformation between *x* and *z*, in utility terms, must be equal to one

$$\frac{\partial U_i}{\partial z_s} \frac{dz_s^*}{dY_s} = \frac{\partial U_i}{\partial x_i}$$

(ii) both other players' labor allocations to player *s*'s plot must be independent of player *i*'s labor allocation

$$\frac{dN_s^s}{dN_i^s} = \frac{dN_{-s}^s}{dN_i^s} = 0$$

(iii) player s's labor allocation to player *i* must also be independent of player *i*'s labor allocation to her plot.

$$\frac{dN_s^{i^*}}{dN_i^s} = 0$$

However, from Corollary 1, we know that player s will provide labor on player *i*'s plot as long as there is some degree of altruism $\frac{\partial U_s}{\partial z_i} > 0$. Therefore, the allocation of player *i*'s labor cannot be efficient. ▪

Section 2. Cooperative Agreement between Players i and j

Production decisions are separable, so labor allocations are determined independent of the utility maximization problem. For simplicity, assume that the participation constraints are not binding for both players such that the joint maximization problem becomes:

$$\max_{x_j, z_i, z_j} \lambda U_i(\cdot) + (1-\lambda) U_j(\cdot) \quad \text{where} \;\; x_i = Y_i + Y_j - p_i z_i - p_j z_j - x_j$$

with first order conditions

$$(1-\lambda)\frac{\partial U_j}{\partial x_j} - \lambda \frac{\partial U_i}{\partial x_i} = 0$$
$$\lambda \frac{\partial U_i}{\partial z_i} + (1-\lambda)\frac{\partial U_j}{\partial z_i} - \lambda \frac{\partial U_i}{\partial x_i} p_i = 0$$
$$\lambda \frac{\partial U_i}{\partial z_j} + (1-\lambda)\frac{\partial U_j}{\partial z_j} - \lambda \frac{\partial U_i}{\partial x_i} p_j = 0$$

Let D denote the determinant of the Hessian, and define its elements as

$$s_{11} = (1-\lambda)\frac{\partial^2 U_j}{\partial {x_j}^2}$$
$$s_{12} = (1-\lambda)\frac{\partial^2 U_j}{\partial x_j \partial z_i} - \lambda \frac{\partial^2 U_i}{\partial x_i \partial z_i}$$
$$s_{13} = (1-\lambda)\frac{\partial^2 U_j}{\partial x_j \partial z_j} - \lambda \frac{\partial^2 U_i}{\partial x_i \partial z_j}$$
$$s_{22} = \lambda \frac{\partial^2 U_i}{\partial {z_i}^2} + (1-\lambda)\frac{\partial^2 U_j}{\partial {z_i}^2} - \lambda \frac{\partial^2 U_i}{\partial x_i \partial z_i} p_i$$
$$s_{23} = \lambda \frac{\partial^2 U_i}{\partial z_i \partial z_j} + (1-\lambda)\frac{\partial^2 U_j}{\partial z_i \partial z_j} - \lambda \frac{\partial^2 U_i}{\partial x_i \partial z_j} p_i$$
$$s_{33} = \lambda \frac{\partial^2 U_i}{\partial {z_j}^2} + (1-\lambda)\frac{\partial^2 U_j}{\partial {z_j}^2} - \lambda \frac{\partial^2 U_i}{\partial x_i \partial z_j} p_j$$

Define $\frac{\partial U_i}{\partial z_j} = \theta_i \frac{\partial U_i}{\partial z_i}$. Then for $\theta > 0$ and all goods are separable in utility,

$$\frac{dz_i}{d\theta_i} = \frac{1}{D}\lambda \frac{\partial U_j}{\partial z_j}(s_{11}s_{23} - s_{12}s_{13}) = 0 \; and \; \frac{dx_j}{d\theta_i} = -\frac{dx_i}{d\theta_i} = -\frac{1}{D}\lambda \frac{\partial U_j}{\partial z_j}(s_{12}s_{23} - s_{13}s_{22}) = 0$$

Section 3. Coalition-Proofness

The equilibrium is coalition-proof (Bernheim, Peleg and Whinston, 1987) if coalitions, once formed, cannot be re-formed for some minimum number of periods such that the gain to deviating is not Pareto-improving for any coalition. Additionally, the following condition must hold, where ^ denotes cooperation between *i* and j, $'$ denotes the fully non-cooperative outcome, and ˜ denotes cooperation between *i* and *k* and between *j* and *k*, respectively.

$$(\hat{V}_i - V_i') + (\hat{V}_j - V_j') > [(\tilde{V}_i - V_i') + (\tilde{V}_k - V_k')] + [(\tilde{V}_j - V_j') + (\tilde{V}_k - V_k')]$$

This ensures that the husband cannot simultaneously offer both wives cooperative agreements that dominate the agreement between co-wives.